STANFORD UNIVERSITY
Department of Aeronautics and Astronautics

December 1975

SUBJECT: Semiannual Progress Report, covering the period June - November 1975, on NASA Research Grant NGL 05-020-243, "Refined Methods of Aeroelastic Analysis and Optimization," Holt Ashley, Principal Investigator

TO: Mr. R. V. Doggett, Jr., NASA Langley Research Center, Technical Monitor, and NASA Scientific and Technical Information Service.

This report will summarize progress on the subject grant since the Semiannual Report submitted in May 1975. Two Ph. D. theses have appeared subsequent to that date:

1. Vepa, Ranjan, "Finite State Modeling of Aeroelastic Systems," Ph. D. Dissertation, Department of Applied Mechanics, Stanford Univ., May 1975.

2. Johnson, Erwin, H., "Optimization of Structure Stochastic Excitation," Ph. D. Dissertation, Department of Aeronautics and Astronautics, Stanford Univ., June 1975.

A third thesis by Mr. Paulo Rizzi on an optimality criterion approach to minimum-weight design of large structures under various types of constraints, will be completed during the first quarter of 1976. Its contents are summarized briefly below but not elaborated, in anticipation that the thesis in extenso will be available shortly.

Several steps have been taken recently toward publication of the Vepa thesis, and some material derived therefrom. First, it has been recommended in two letters to cognizant NASA Langley personnel that Dr. Vepa's work, with a few minor corrections, merits distribution as a low-number Contractor Report. Because replies have not yet been received to these communications, the P. I. is not in a position to comment on the disposition of this proposal. Since Dr. Vepa is currently employed at Langley on a Post-Doctoral appointment, and should he be willing to cooperate, it is noted that his presence might facilitate the editing of such a C. R.

Dr. Rizzi has submitted an extended abstract of his thesis for presentation at the AIAA/ASME 17th Structures, Structural Dynamics and Materials Conference, May 1976. Also submitted to this Conference was a paper by Johnson, Rizzi, Segenreich and Ashley, entitled "Optimization of Continuous One-Dimensional Structures Under Steady Harmonic Excitation." Some contents of the latter paper are summarized in the second section of this report.

Although receipt of the two 17th SDM submissions has been acknowledged, acceptance cannot be reported; notification thereof was promised by Decemer 29, 1975, but has not yet been received. Whether or not they are delivered at a technical meeting, however, both papers will ultimately be submitted to AIAA Journal or Journal of Aircraft.

(NASA-CR-146020) REFINED METHODS OF AEROELASTIC ANALYSIS AND OPTIMIZATION (Stanford Univ.) 16 p
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Inasmuch as the Johnson and Vepa theses are already in the hands of the Technical Monitor, it seems unnecessarily duplicative to detail their contents here. Since mid-1975, major efforts under this grant have otherwise been denoted to the following areas of investigation:

Optimization Algorithm for Large Structures Subject to Various Constraints: The optimal design of structures is an iterative trail-and-error process, each iteration consisting of two steps. An analysis of the structure at the current design, followed by a redistribution of material.

This investigation has been focused on the redesign cycle. An optimality criterion algorithm was developed to solve the following problem: Minimize the weight

$$W = \sum_{i}^{N} a_i x_i \, ,$$

subject to behavioral constraints

$$g_i(\underset{\sim}{x}) \leq 0 \qquad j = 1, \ldots M,$$

and to size constraints

$$(x_i)_{min} \leq x_i \leq (x_i)_{max} \qquad i = 1, \ldots N \, .$$

The weight is assumed to depend linearly on each of the design variables x_i, which represent member thicknesses, cross-sectional areas, and the like.

The Kuhn Tucker conditions are used to derive the redesign equation

$$x_i^{(y+1)} = \begin{cases} c_i x_i^{(\nu)} & \text{if} \quad (x_i)_{min} \leq c_i x_i^{(\nu)} \leq (x_i)_{max} \\ (x_i)_{min} & \text{if} \quad c_i x_i^{(\nu)} < (x_i)_{min} \\ (x_i)_{max} & \text{if} \quad c_i x_i^{(\nu)} > (x_i)_{max} \end{cases}$$

where

$$c_i = \alpha - \frac{(1-\alpha)}{a_i} \sum_{j=1}^{M} \lambda_j \frac{\partial g_j}{\partial x_i} \, .$$

Here $-1 < \alpha < 1$ is a relaxation parameter to improve convergence, and

$$\lambda_j \begin{cases} = 0 & \text{if} \quad g_j < 0 \\ \geq 0 & \text{if} \quad g_j = 0 \, . \end{cases}$$

A new and efficient method was developed for the calculus of the Lagrange multipliers λ_j.

The algorithm is shown to be fast (efforts spent in the redesign cycle are minimal), efficient (a small number of steps are needed for convergence of the overall design process), and general (no restrictions are imposed on the type of behavioral constraints).

Numerous applications were carried, including wing structures subject to requirements on strength, stiffness, frequency and flutter.

Some Typical Results: Consider the rectangular wing of Fig. 1. The structure is subject to two loading conditions and constraints are imposed on the maximum stresses, tip displacements, first natural frequency and flutter speed (see Table 1). Final material distributions for four independent problems are given in Figs. 2 and 3. Iteration histories are given in Fig. 4. It is interesting to note that the optimal weight under static constraints alone is practically the same when a flutter constraint is also imposed, although the material distribution is quite different. It appears that, because the active static constraints are very "flat" at the optimum, a material redistribution which increases the flutter speed is possible without significantly increasing the weight.

Otpimization of Continuous One-Dimensional Structures Under Steady Harmonic Excitation: Figure 4 shows a bar with continuously varying cross section, fixed at $x = 0$ and subject to a harmonically varying axial load $Pe^{i\omega_e t}$ applied at the end $x = \ell$. The problem is to minimize the total weight of the rod

$$\min W = M_T + \rho \int_0^{\ell} A(x)dx$$

where M_T is a variable tip mass. Constraints are imposed on the maximum peak stress and on the minimum-admissible cross sectional area. The same mathematics apply to a thin-walled cantilever rod excited in torsion.

Because of the disjoint property of the feasible region (see E. Johnson thesis), only a partial solution to the problem is possible. The use of optimal control methods has permitted the finding of two distinct optimal solutions:

a) First-Mode Solution $(\omega_e < \omega_1)$

Here the optimal bar is vibrating in phase with the applied load, and the excitation frequency is smaller than the first natural frequency. It is seen (Fig. 6) that the fully-stressed solution* is optimal for small excitation frequencies

$$(\alpha \leqslant 1.091, \quad \alpha = \omega_e \ell \frac{\rho}{2E}).$$

*That is, a solution in which each station along the bar is working to the peak allowable stress.

For larger excitation frequencies the optimal bar is unconstrained from the root up to $x/\ell = \gamma$ and constrained to peak stress thereafter. This can be seen in Fig. 7, where the optimal area distributions are plotted and compared with finite element solutions obtained by the optimality criteria algorithm summarized above. It should be noted that the minimum area occurs at the tip, with value $A = P/\sigma_{max}$ and that for the optimal solution the tip mass vanishes, even though the problem statement allows for its presence.

b) Second Mode Solution $(\omega_e > \omega_1)$

Here the optimal rod is vibrating 180^o out of phase with the applied load, and the excitation frequency is bigger than the first natural frequency. For this case, in order to satisfy the boundary conditions, a tip mass will be necessary and the specified minimum area will be reached. The root portion has active stress constraints, and the tip portion is at the minimum area. Figure 8 shows a tyipcal area distribution, which is compared with the results using two finite element representations. For the continuous solution, the tip mass is represented by

$$A_T = \frac{M_T}{\rho \ell}$$

The nondimensional optimal volume as a function of the excitation frequency parameter is represented in Fig. 9. It is seen that, for a given minimum thickness parameter the second mode solution is better than the first at sufficiently high excitation frequencies.

The thesis discussing this study as well as the previous one is in the final writing process and will appear in March 1976:

Paulo Rizzi, "The Optimization of Structures with Complex Constraints Via a General Optimality Criteria Method," Ph. D. Thesis, Stanford University, March 1976.

Dynamics of Hingeless Rotor Blades Without and With Aerodynamic Loading: Mr. William Boyd has begun an effort to extend and apply some work of Peters (NASA TMX-62,299, 1974), aimed at using the method of matched asymptotic expansions to analyze the dynamics of cantilever rotating blades of various kinds. The small parameter for the expansions is generally the ratio of blade bending rigidity to centrifugal tension. The "outer" expansion, over portions of the blade not too close to the ends, is made assuming the dominant forces to involve the inertia loads due to vibration and the centrifugal "stiffening" forces. The root "boundary layer" consists of a small zone where bending rigidity and centrifugal effects predominate, whereas bending and vibratory inertia are most prominent near the free tip.

All of Peters' results (torsion, in-plane bending out-of-plane bending) have been examined and verified. One question still under examination is whether it is always essential to include the tip "boundary layer" in order to obtain accurate estimates for natural frequencies and modes of free vibration. Aerodynamic terms of quasi-steady, strip type have been incorporated into the equations of motion. Although this extension obviously complicates the analyses — and will do so more severely when the unsteadiness needed for flutter prediction is accounted for — it nevertheless preserves the basic conditions underlying the matched-expansion scheme. That is, the relationships remain unchanged among the highest-derivative terms. Since this work is still in its early stages, no numerical results can yet be reported.

Mr. Boyd is also engaged in the application of expansion methods to verify and simplify an earlier analysis by Petre of the influence of drag on the divergence instability of large-aspect-ratio, unswept, cantilever wings. A regular rather than a singular perturbation is encountered here. Preliminary indications are that considerable computational simplification is achieved by comparison with modal or finite-element solution of the complete problem.

Aeroelastic Phenomena Involving Active Controls: Under NASA NSG 4002, entitled "Theoretical and Experimental Investigations in the Control of Aircraft," the Department's Guidance and Control Lab. have been collaborating with the P. I. on applications of modern automatic control technology to the improvement of aeroelastic stability of wings. In addition to theoretical analyses, this activity has consisted of the construction of a very high-quality "typical section" flutter model, now installed in the Stanford 0.5-meter low-speed wind tunnel. This model idealizes the bending and torsion degrees of freedom of a straight wing in a manner familiar to aeroelasticians, with adjustable spring restraints on each freedom, but it also incorporates provisions for force and torque actuation at a simulated elastic axis. Accelerometers measure the motions, and their signals can be processed (in a digital or analog fashion) so as to drive the actuators in accordance with any desired feedback law. The model has already been tested "open-loop" and reproduces quite well the predicted flutter performance.

The finite-state idealizations of aerodynamic transfer functions, developed by Vepa in his thesis, were incidentally used in the model development.

As part of the present grant activity, the P. I. is structuring a program of theoretical research to complement the F. R. C. project, and Mr. John Edwards of that project is already actively involved. In cooperation with Prof. J. V. Breakwell of Guidance and Control, a fundamental study is under way of exactly how the transcendental relations from unsteady aerodynamic theory affect the transfer function used in control system design. For instance, the Theodorsen function of two-dimensional, incompressible aerodynamics (together with its generalizations for arbitrary small motions) had formerly been thought to augment ordinary mechanical systems by an "infinite" number of degrees of freedom or states. It now appears, however, that a simple iterative scheme for control-logic design may be devised wherein no state augmentation at all is required. Whether this approach can be generalized to 3-D and/or compressible flow is still being investigated.

As mentioned in the July 1975 continuation proposal, a small program along the foregoing and related lines is still under development.

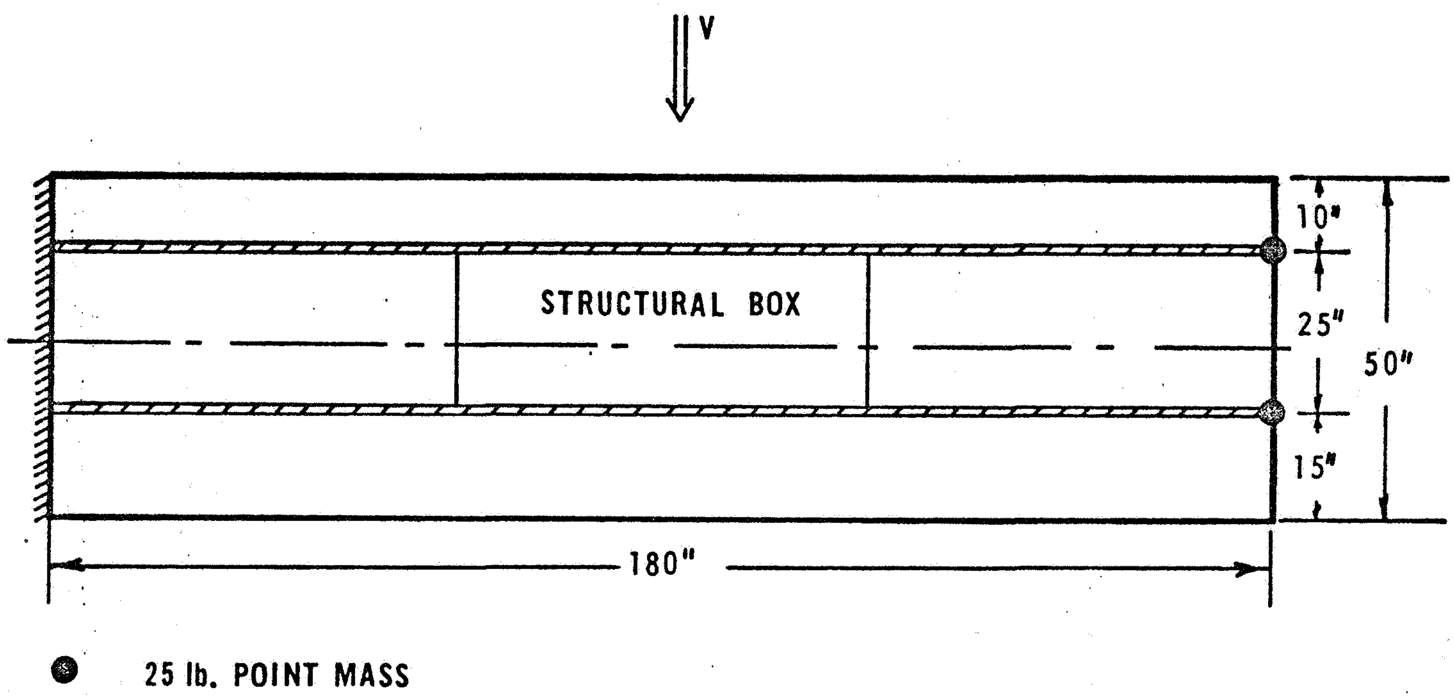

Fig. 1. Rectangular Wing

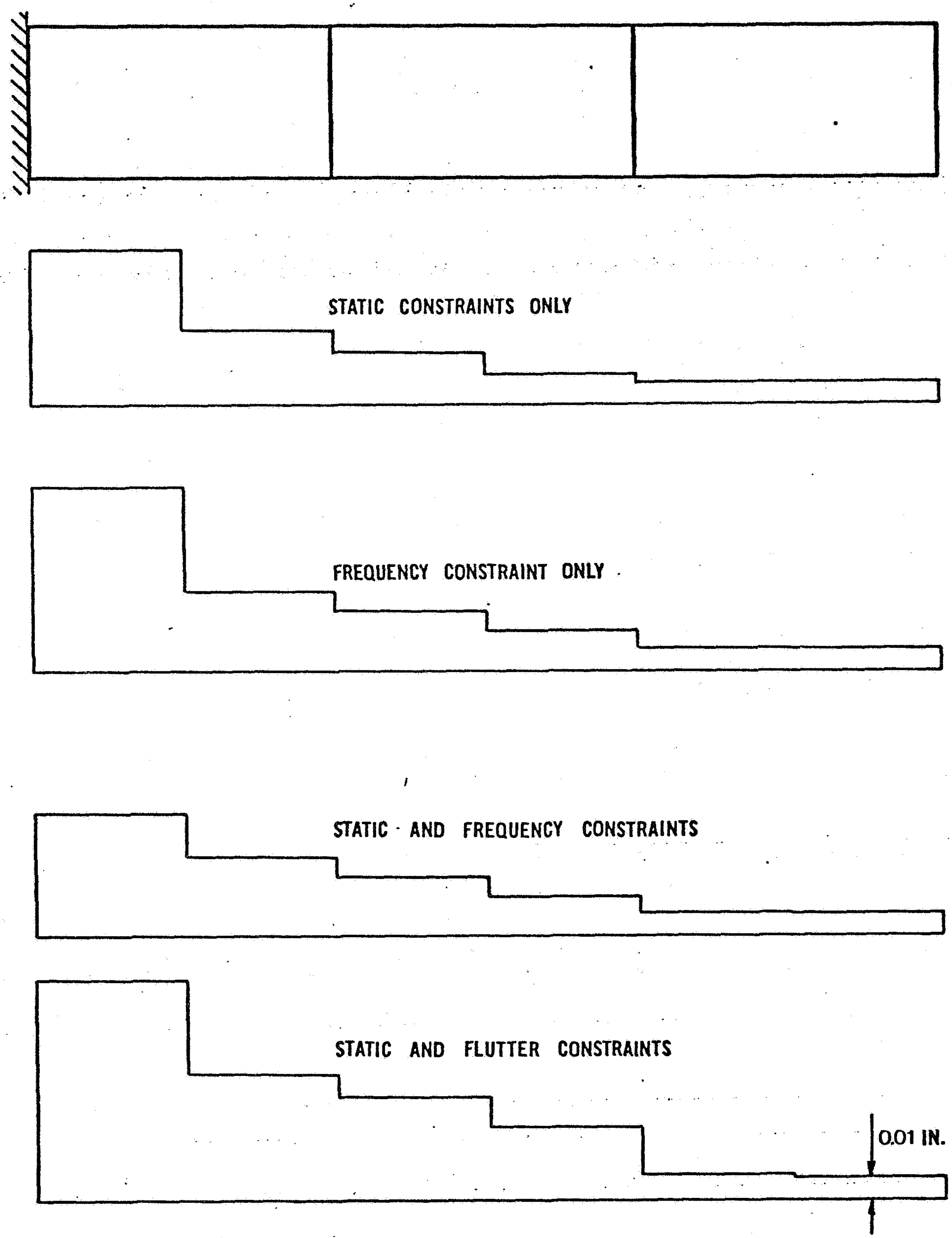

Fig. 2. Final Skin Panel Thickness Distribution. Rectangular Wing.

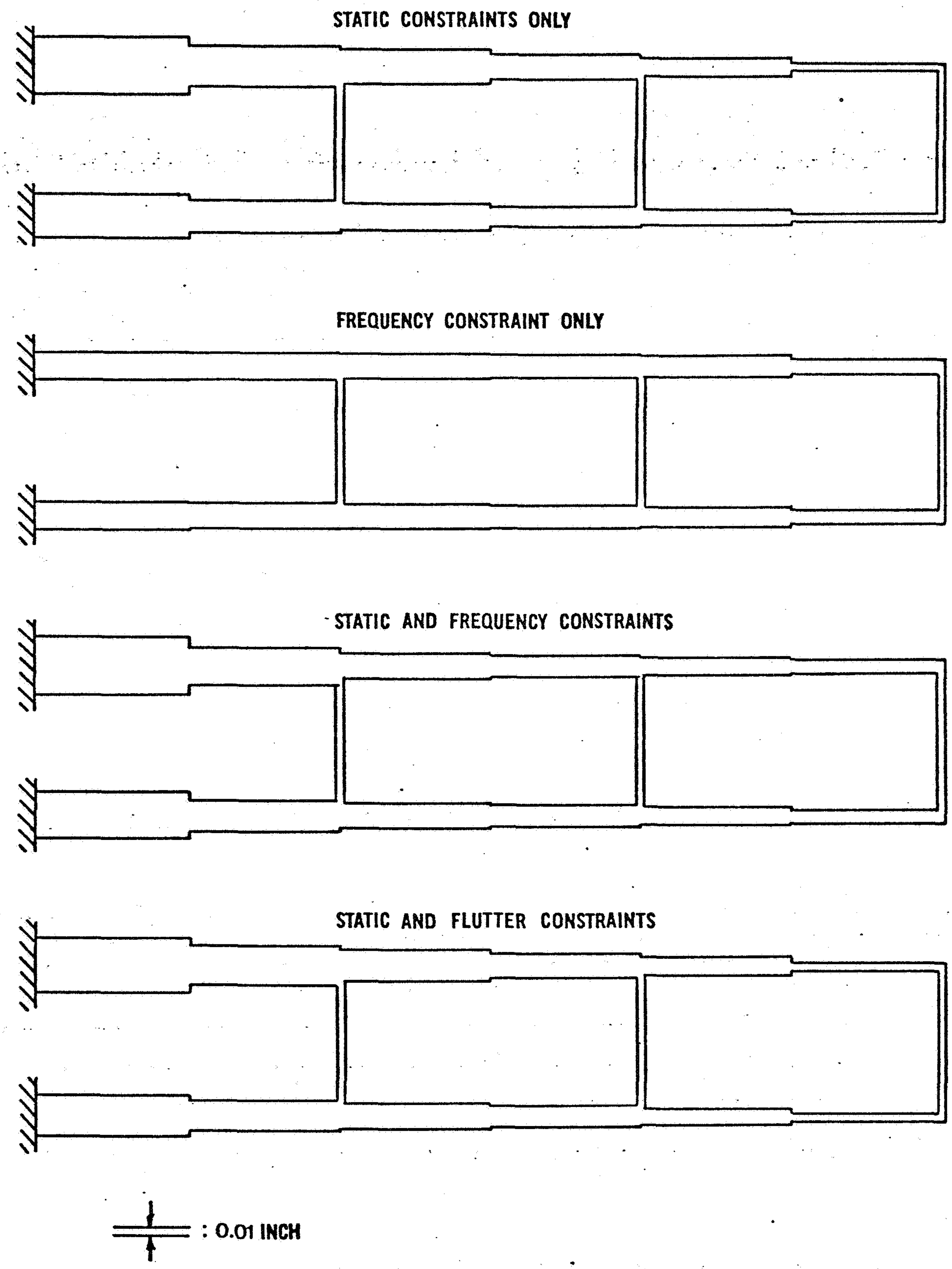

Fig. 3. Final Web Thickness Distribution. Rectangular Wing.

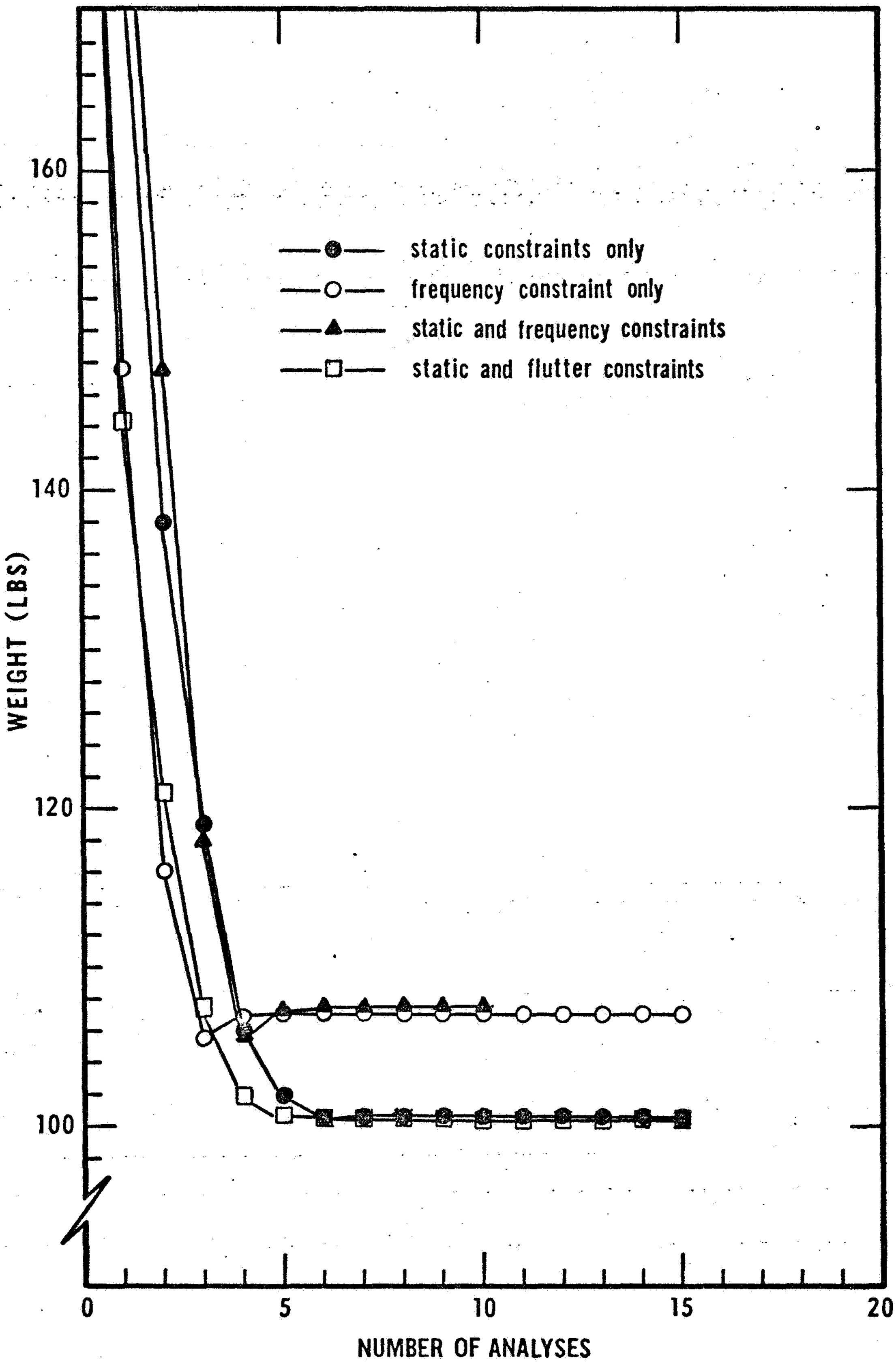

Fig. 4. Iteration Histories for Rectangular Wing Problems.

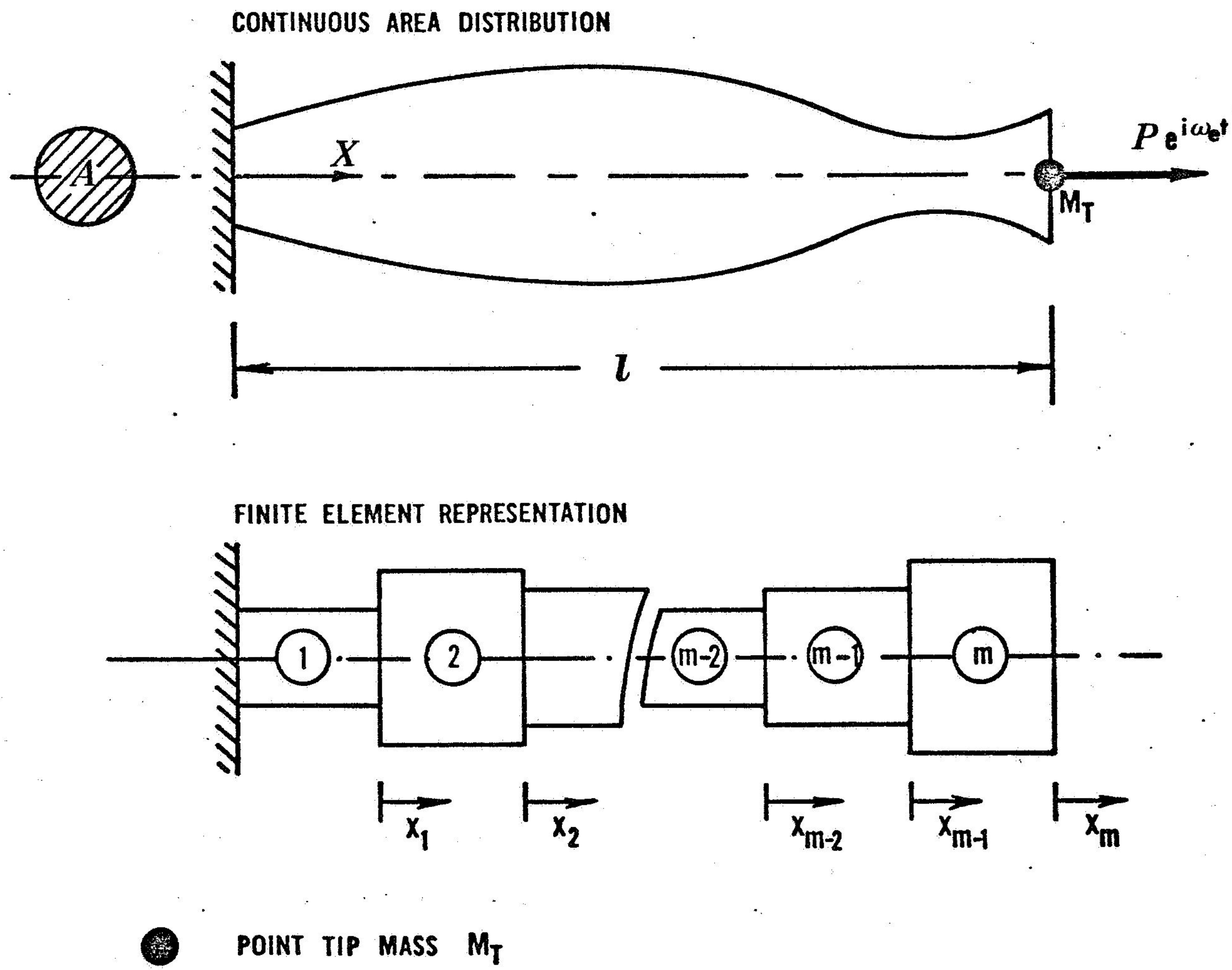

Fig. 5. Cantilever Rod with An Axial Sinusoidal Load at the Tip, Shown in Continuous and Finite-Element Representations.

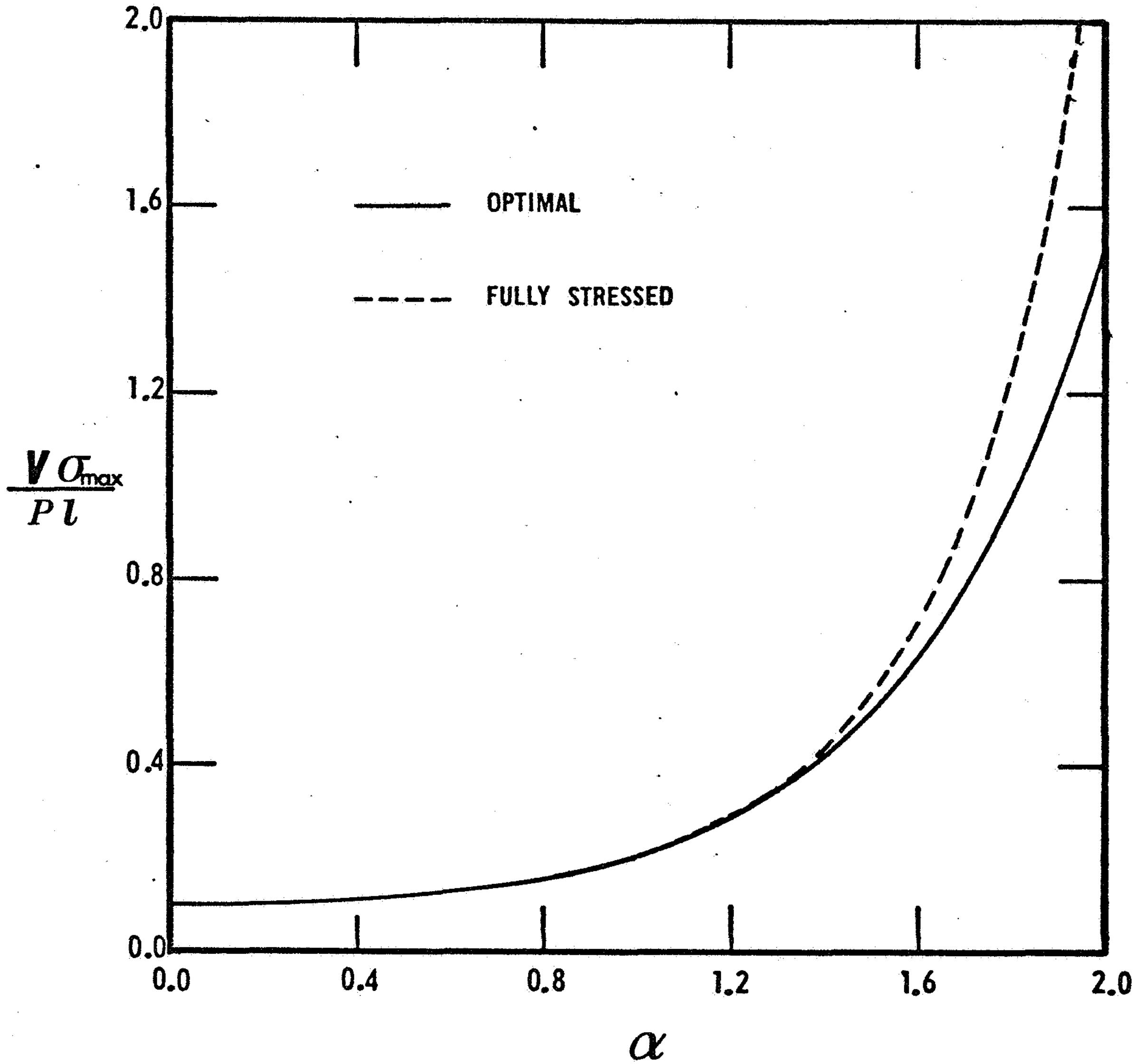

Fig. 6. Comparison of the Volumes of Optimal and Fully Stressed Solutions as a Function of Excitation Frequency Parameter α, When the First Natural Frequency is Greater than the Excitation Frequency (First Mode).

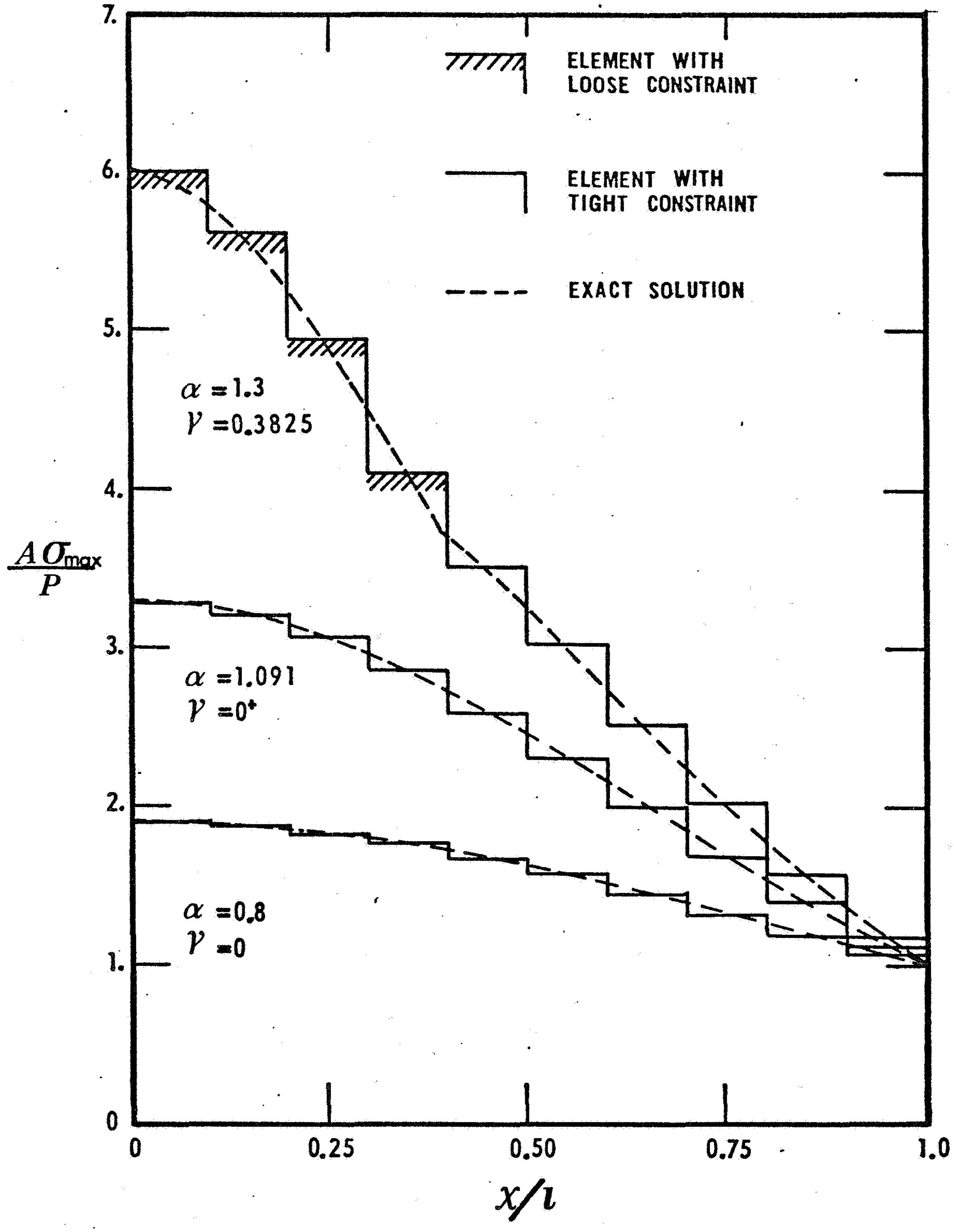

Fig. 7. Comparison of the Area Distributions for the First-Mode ($\omega_e < \omega_1$) Continuous and Finite-Element Solutions.

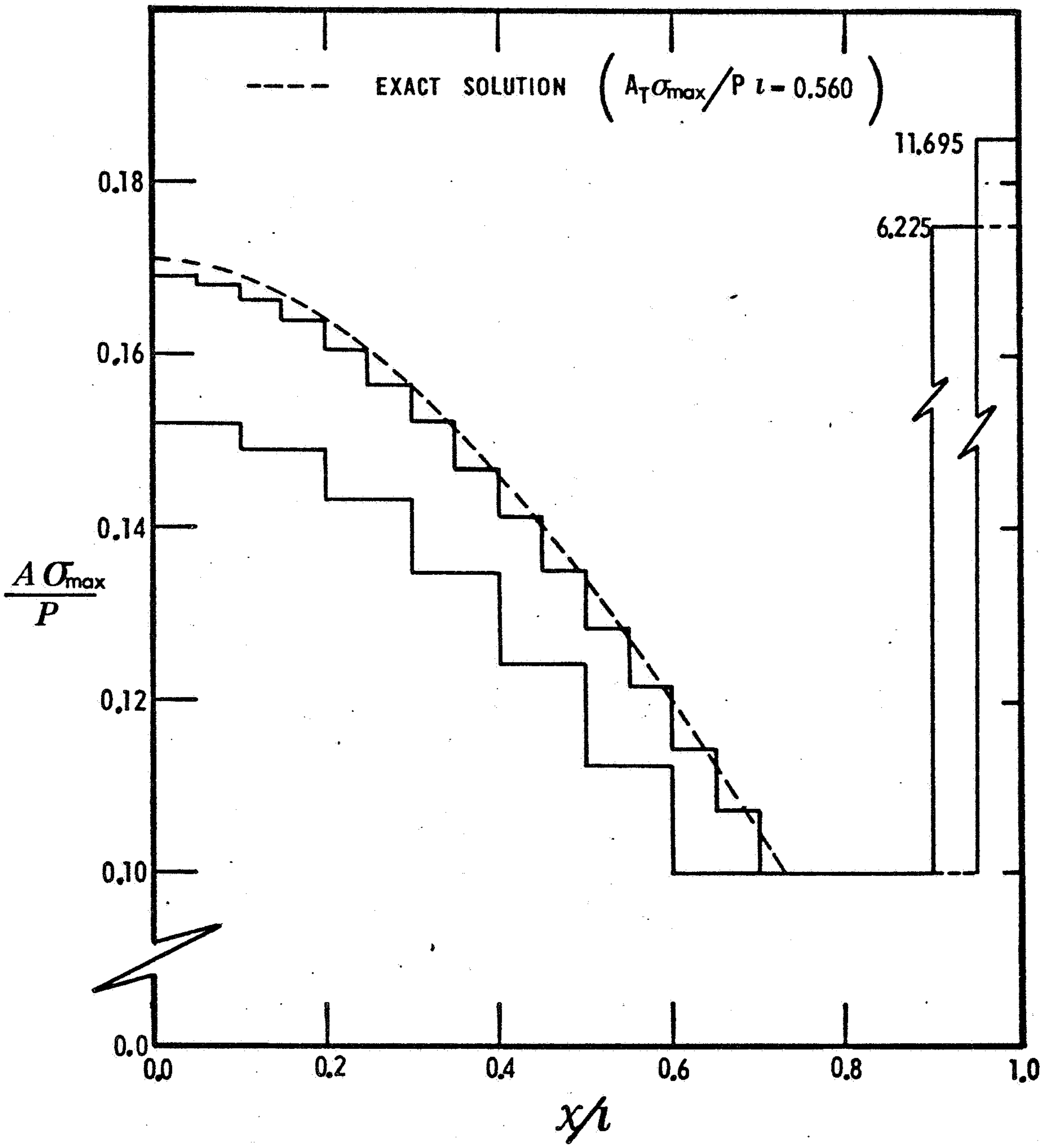

Fig. 8. Comparison of the Area Distributions for Continuous and Finite-Element Solutions with A First Natural Frequency that is Smaller than the Excitation Frequency (Second Mode) ($\alpha = 1$).

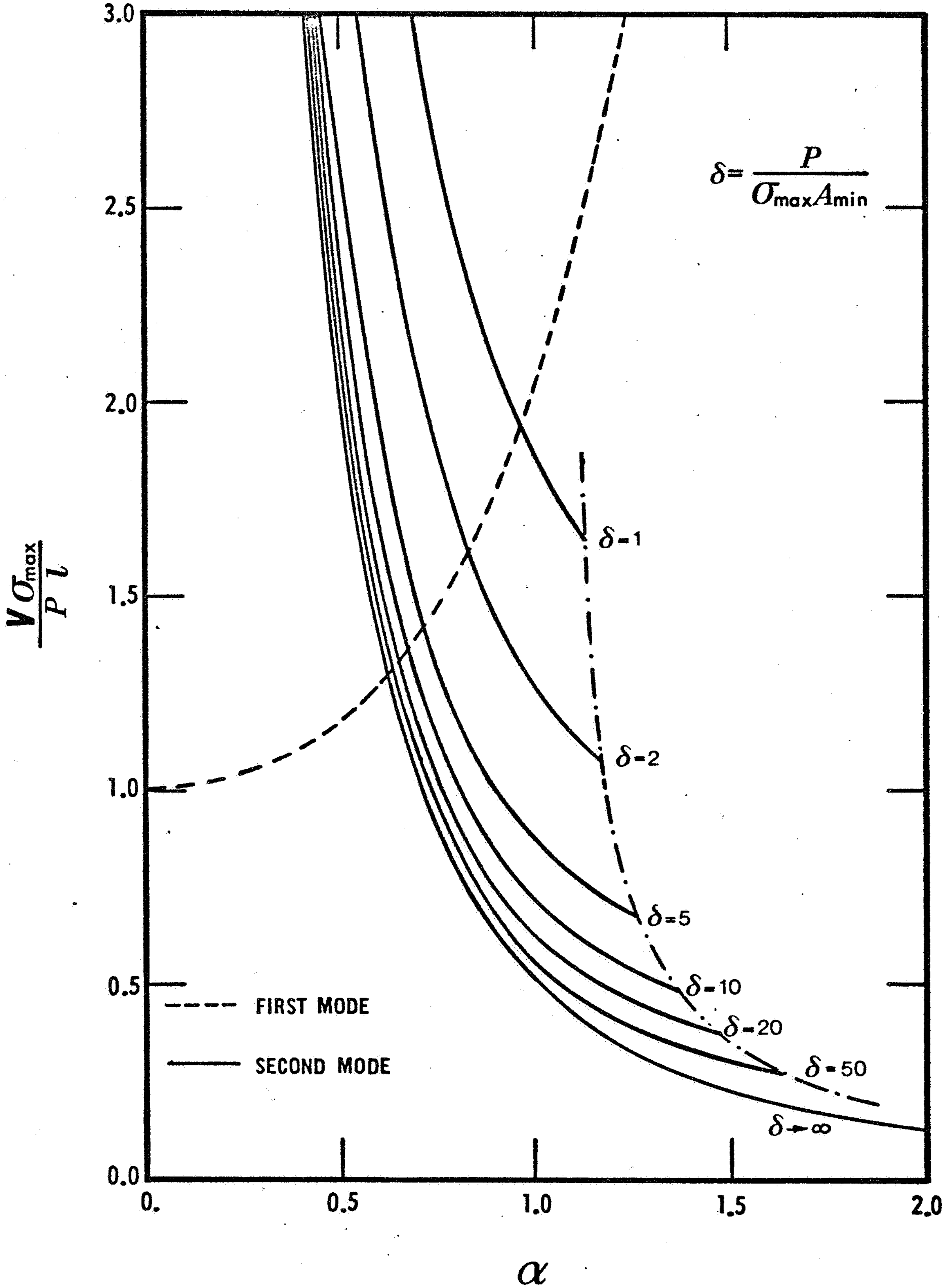

Fig. 9. Comparison of the Volumes of First and Second Mode Solutions as A Function of Excitation Frequency Parameter α.

Table 1. Load Data, Material Properties and Constraint Information

1) Load Data

Node	Load Condition 1 (lbs)	Load Condition 2 (lbs)
3	165.8	299.6
4	414.4	374.5
5	142.6	257.6
6	356.4	322.1
7	131.1	236.6
8	327.4	295.8
9	117.7	212.7
10	294.3	265.9
11	97.8	176.8
12	244.5	220.9
13	38.1	68.9
14	95.3	86.1

2) Material Properties

Specific Weight $\rho = 0.100$ lbs/in^3

Hodulus of Elasticity $E = 10\times10^6$ psi

Poisson's Ratio $\nu = 0.3$

3) Constraint Information

Stress Upper Limit $\sigma^{(U)} = 25,000$ psi

Stress Lower Limit $\sigma^{(L)} = -25,000$ psi

Displacement Limit $D_{max} = 11.0$ in

Fundamental Frequency Lower Limit $\omega^* = 3.83$ cps

Flutter Velocity Lower Limit $V_F = 795$ ft/s ($\rho_{air} = 1.0\times10^{-7}$ lb. sec^2/in^4)